9. Schulübung

·) Welcher Punkt der Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ hat vom rechten Brennpunkt den kleinsten Abstand?

$a = 5,\ b = 4,\ e = \sqrt{a^2 - b^2} = 3 \Rightarrow F_2 = (3|0)$

HB: $|\overrightarrow{F_2X}| \to \text{Min.}$

ND: $\frac{x^2}{25} = 1 - \frac{y^2}{16} \quad | \cdot 25$

$x^2 = 25 \cdot \left(1 - \frac{y^2}{16}\right) \quad | \sqrt{\ }$

$x = 5 \cdot \sqrt{1 - \frac{y^2}{16}}$

HB: $d(y) = \left| \left(5 \cdot \sqrt{1 - \frac{y^2}{16}} - 3 \right) \right| = \sqrt{25 \cdot \left(1 - \frac{y^2}{16}\right) - 30 \cdot \sqrt{1 - \frac{y^2}{16}} + 9 + y^2} \to \text{Min}$
y

$\bar{d}(y) = 25 - \frac{25}{16} y^2 - 30 \cdot \sqrt{1 - \frac{y^2}{16}} + 9 + y^2 =$

$= 34 - \frac{9}{16} y^2 - 30 \cdot \sqrt{1 - \frac{y^2}{16}}$

$\bar{d}'(y) = -\frac{9}{8} y - 30 \cdot \frac{-\frac{y}{8}}{2 \cdot \sqrt{1 - \frac{y^2}{16}}} = -\frac{9}{8} y + \frac{15y}{8 \cdot \sqrt{1 - \frac{y^2}{16}}} = 0$

$\frac{15y}{8 \cdot \sqrt{1 - \frac{y^2}{16}}} = \frac{9y}{8} \quad | \cdot 8\sqrt{1 - \frac{y^2}{16}}$

$15y = 9y \cdot \sqrt{1 - \frac{y^2}{16}} \quad | ^2$

$225y^2 = 81y^2 \cdot \left(1 - \frac{y^2}{16}\right)$

$225y^2 = 81y^2 - \frac{81y^4}{16}$

$\frac{81y^4}{16} + 144y^2 = 0$

$y^2 \left(\frac{81}{16} y^2 + 144\right) = 0 \Rightarrow y_1 = 0 \Rightarrow x = 5$

$y_2 = \pm \sqrt{-\frac{2304}{81}} \notin \mathbb{R}$

$X = (5|0)$