

4. Schülübung

Extremwertaufgaben

3.110) x und y : $xy^2 \rightarrow \text{Max.}$

Problem: Funktion mit 2 Variablen $f(x, y) = xy^2$

Lösung: 2. Bedingung $x + y = 93$

(1) HB: $f(x, y) = xy^2 \rightarrow \text{Max}$ (Hauptbedingung ist immer das, was max. oder min. werden soll)

(2) NB: $x + y = 93 \quad | -y$
 $x = 93 - y$ (aus Nebenbedingung immer eine Variable ausdrücken)

(3) NB \rightarrow HB: $f(y) = (93 - y)y^2 = 93y^2 - y^3$

$$f'(y) = 186y - 3y^2$$

$$f''(y) = 186 - 6y$$

$$186y - 3y^2 = 0 \quad | :3$$

$$62y - y^2 = 0 \quad | y \text{ herausheben}$$

$$y(62 - y) = 0 \quad | \text{Produkt-0-Regel}$$

$$\underline{y_1 = 0} \quad \vee \quad 62 - y = 0$$

$$\underline{y_2 = 62}$$

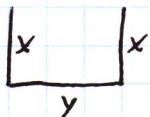
[4] $f''(0) = 186 \Rightarrow \text{Min.} \Rightarrow \text{ungültig}$

$f''(62) = -186 \Rightarrow \text{Max.} \checkmark$

(5) Lösung \rightarrow NB: $\underline{x = 93 - 62 = 31}$

[6] Ergebnisse \rightarrow HB: $f(31; 62) = 119164$

3.116)



(1) HB: $A(x,y) = xy \rightarrow \text{Max.}$

(2) NB: $2x + y = 40$

$$y = 40 - 2x$$

(3) NB \rightarrow HB: $A(x) = x \cdot (40 - 2x) = 40x - 2x^2$

$$A'(x) = 40 - 4x$$

$$A''(x) = -4 \quad (\Rightarrow \text{Max.})$$

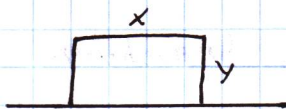
$$40 - 4x = 0$$

$$\underline{x = 10}$$

(5) $y = 40 - 20 = 20$

Maximale Fläche: $x = 10$; $y = 20$

3.117)



HB: $A(x,y) = xy - \text{Max.}$

NB: $x + 2y = 20$

$$x = 20 - 2y$$

NB \rightarrow HB: $A(y) = (20 - 2y) \cdot y = 20y - 2y^2$

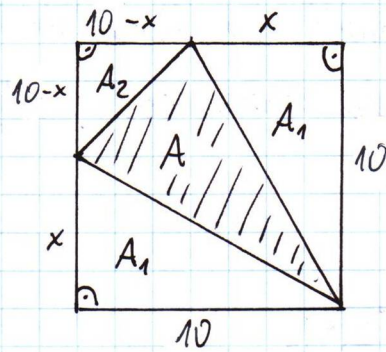
$$A'(y) = 20 - 4y$$

$$A''(y) = -4 \Rightarrow \text{Max}$$

$$20 - 4y = 0$$

$$\left. \begin{array}{l} \underline{y = 5} \\ \underline{x = 10} \end{array} \right\} \text{Maximale Fläche}$$

3.119)



Rechnen mit $a = 10$

$$A = A_{\square} - 2 \cdot A_1 - A_2$$

$$A_1 = \frac{10x}{2}$$

$$A_2 = \frac{(10-x)^2}{2} = \frac{100 - 20x + x^2}{2}$$

$$A = 100 - 10x - \frac{100 - 20x + x^2}{2} = \frac{200 - 20x - 100 + 20x - x^2}{2}$$

$$A(x) = \frac{100 - x^2}{2} = 50 - \frac{1}{2}x^2$$

$$A'(x) = -x$$

$$A''(x) = -1 \Rightarrow \text{Max.}$$

$$-x = 0$$

$$\underline{\underline{x = 0}}$$

Dreieck = halbes Quadrat

